ANALYTIC CALCULATION OF TEMPERATURES DURING GRINDING

V. I. Ostrovskii and V. V. Kazachek

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The problem of determining the quasi-steady temperature field in a part during grinding is examined allowing for periodic thermal effects; the differential heat-conduction equation is analyzed with boundary layer theory.

The reliability and lifetime of machines and tools are determined mainly by the structure and finish of the component working surfaces, which in turn depend on the thermal conditions during the final operation of fabrication, i.e., grinding. The question of a theoretical investigation of the temperature field during grinding is receiving a good deal of attention.

References [1, 2] suggest an analytical method of solving this problem; it is based on the solution of the heat-conduction equation with natural boundary conditions of the fourth kind; the one-dimensional heat-conduction equation was considered without allowing for the intermittent nature of the heat source. The present paper considers the problem of determining the temperatures of a rotating component, with account taken of the periodic thermal effect.

The quasi-steady temperature state of a rotating body in a fixed system of coordinates can be described by the heat-conduction equation

$$\frac{\partial T}{\partial \varphi_1}\omega = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi_1^2}\right) \tag{1}$$

with the boundary conditions

$$\lambda \left.\frac{\partial T}{\partial r}\right|_{\substack{r=R \\ -\varphi_{10}\leqslant\varphi_1\leqslant\varphi_{10}}} = q(\varphi_1) + \alpha_k T, \tag{2}$$

$$\lambda \left.\frac{\partial T}{\partial r}\right|_{\substack{r=R \\ \varphi_{10}<\varphi_1<2\pi-\varphi_{10}}} = \alpha T, \tag{3}$$

where T is finite.

Transforming to dimensionless form, we obtain

$$\frac{\partial T}{\partial \varphi} N = \frac{\partial^2 U}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial U}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 U}{\partial \varphi^2}, \tag{4}$$

where

$$N = \omega R^2/2a,\quad \rho = r/R,\quad U(\rho,\varphi) = 2\pi\alpha T(r,\varphi)/q_0,$$

$$\frac{\partial U}{\partial \rho} = \begin{cases} -2\pi\beta f(\varphi), & -\varphi_0 \leqslant \varphi \leqslant \varphi_0,\ \rho = 0, \\ \beta U, & \varphi_0 < \varphi < 2\pi - \varphi_0,\ \rho = 0, \end{cases}$$

$$\left.\frac{\partial U}{\partial \rho}\right|_{\rho\to\infty} \to 0,$$

$$U(\rho,\varphi) = U(\rho, \varphi + 2\pi n),\quad n = 0, 1, \ldots, k,$$

$$f(\varphi) = q(\varphi)/q_0,$$

where $\beta = \alpha R/\lambda$, and the parameter N, corresponding to the Peclet number Pe, lies in the range $[10^3-10^5)$ for the majority of practical cases of grinding.

Thus, in an equation of the second order, we have a large parameter N for the first derivative. This indicates the presence of a "certain thermal boundary" layer in which a noticeable temperature change occurs. As we move out from the boundary layer, as Eq. (4) indicates,

$$\frac{\partial U}{\partial \varphi} = \frac{1}{N}\left(\frac{\partial^2 U}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial U}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 U}{\partial \varphi^2}\right) \to 0, \tag{4a}$$

i.e., T = const.

We assume that the dimensionless thickness of the "boundary" layer is δ; we estimate the order of magnitude of the quantities appearing in Eq. (4)

$$\text{a)}\ \frac{\partial U}{\partial \varphi} N \sim \frac{U}{2\pi} N,\quad \text{b)}\ \frac{\partial^2 U}{\partial \rho^2} \sim \frac{U}{\delta^2},$$

$$\text{c)}\ \frac{1}{\rho}\frac{\partial U}{\partial \rho} \sim \frac{U}{\delta}\quad \text{d)}\ \frac{1}{\rho^2}\frac{\partial^2 U}{\partial \varphi^2} \sim \frac{U}{(2\pi)^2}.$$

For Eq. (4) not to degenerate into an equation of the form $\partial U/\partial\varphi = 0$, we must have the orders of magnitude of the quantities $(\partial U/\partial\varphi)N$ and $\partial^2 U/\partial\rho^2$ equal, i.e., $UN/2\pi = U/\delta^2$. Then the orders of magnitude of the terms of Eq. (4) will be

$$\text{a)}\ \frac{\partial U}{\partial \varphi} N \sim \frac{U}{2\pi} N;\quad \text{b)}\ \frac{\partial^2 U}{\partial \rho^2} \sim \frac{U}{2\pi} N;$$

$$\text{c)}\ \frac{1}{\rho}\frac{\partial U}{\partial \rho} \sim \frac{U}{2\pi}\sqrt{N};\quad \text{d)}\ \frac{1}{\rho^2}\frac{\partial^2 U}{\partial \varphi^2} \sim \frac{U}{(2\pi)^2}. \tag{4b}$$

It can be seen from Eq. (4b) that only a) and b) will be quantities of the first order for large N, while c) and d) are negligibly small, for example, for $N = 10^4$. Dividing expression a) by N, we see that $\partial U/\partial\varphi \sim 1$; $(1/N)(\partial^2 U/\partial\rho^2)$ will also be ~ 1. We shall multiply by 2π, and obtain

$$\frac{1}{N}\cdot\frac{1}{\rho}\frac{\partial U}{\partial \rho} \sim \frac{\sqrt{2\pi}}{\sqrt{N}} \sim \frac{2.5}{100} \sim 0.025, \text{ and}$$

$$\frac{1}{N}\frac{1}{\rho^2}\frac{\partial^2 U}{\partial \varphi^2} \sim \frac{1}{N\cdot 2\pi} \sim \frac{1}{N\cdot 2\pi} \sim \frac{1}{10000\cdot 6.28}.$$

Thus, we see that in the "boundary" layer the quantities c) and d) are negligibly small in comparison with a) and b). Then in Eq. (4), retaining quantities of the first order, we obtain an equation of the form

$$\frac{\partial^2 U}{\partial \rho^2} - \frac{\partial U}{\partial \varphi} N = 0.$$

The majority of cases involves grinding of bodies whose dimensions are considerably larger than the thermal boundary layer thickness. Thus, for example, the boundary layer during grinding is usually 1 to 2 mm, while the thickness of components being ground is usually 30 to 100 mm. For this reason, instead of the condition $T = \text{const}$, for $y = \delta$, we can apply the condition $\partial T/\partial y|_{y\to\infty} \to 0$ to the boundary-layer thickness, which corresponds to the so-called asymptotic boundary layer.

Calculated Values of Contact and Residual Temperatures

γ	T_{max}, °C	T_{min}, °C
0.35	1380	75
0.87	1131	51
0.43	1580	93
1.29	1260	60

The solution of Eq. (4), obtained with the help of the Green's function, has the form

$$U(\rho,\varphi) = \int_{-\varphi_0}^{\varphi_0} \left[f(\varphi') + \frac{U(\varphi')}{2\pi} \right] \left\{ 1 + \right.$$

$$\left. + \sum_{-\infty}^{\infty} \frac{\beta \exp\left[im(\varphi-\varphi') - (2Nmi)^{1/2}\rho\right]}{\beta + (2Nmi)^{1/2}} \right\} d\varphi'.$$

For the temperature at the surface, we obtain the integral equation

$$U(\varphi) = \int_{-\varphi_0}^{\varphi_0} \Big[1 +$$

$$+ 2\beta \sum_{m=1}^{\infty} \left\{ \beta + (Nm)^{1/2} \cos m(\varphi - \varphi') + \right.$$

$$\left. + (Nm)^{1/2} \sin m(\varphi - \varphi') \right\} \left\{ (\beta + Nm)^{1/2} + Nm \right\}^{-1} \Big] d\varphi'.$$

Because of the small angle of contact (0.5–3°), we can assume that

$$f(\varphi') + \frac{U(\varphi')}{2\pi} = \text{const}.$$

Then,

$$U(\varphi) = C \left\{ 2\varphi_0 + 2\gamma \times \right.$$

$$\left. \times \sum_{m=1}^{\infty} \frac{2\sin m\varphi_0 \left[(\gamma + m^{1/2}) \cos m\varphi + m^{1/2} \sin m\varphi \right]}{m(\gamma^2 + 2\gamma + m^{1/2} + 2m)} \right\}. \quad (5)$$

Here $\gamma = \beta(N)^{-1/2}$. We can find the constant C from the condition at the boundary $\rho = 0$,

$$\int_{-\varphi_0}^{\varphi_0} \frac{\partial U}{\partial \rho} d\varphi = -2\pi\beta \int_{-\varphi_0}^{\varphi_0} f(\varphi)\, d\varphi.$$

The series in Eq. (5) converges slowly; thus to obtain an accuracy of 0.001, we must take $4 \cdot 10^6$ terms of the series. Therefore, we should separate out and sum up the slowly convergent parts of the series. Then Eq. (5) is written in the form

$$U(\varphi) =$$

$$= C \left\{ 2\varphi_0 + 2\gamma \sum_{m=1}^{\infty} \left[\cos m(\varphi - \varphi_0) - \cos m(\varphi + \varphi_0) \right] \times \right.$$

$$\times \frac{1}{2} \left[\frac{1}{m^{3/2}} - \frac{\gamma}{m^2} + \frac{\gamma^2}{2m^{5/2}} - \frac{\gamma^4}{4m^{7/2}} + \right.$$

$$\left. + \frac{\gamma^5(\gamma + 2m^{1/2})}{8m^{7/2}\left(\frac{\gamma^2}{2} + \gamma m^{1/2} + m \right)} \right] +$$

$$+ \left[\sin m(\varphi + \varphi_0) - \right.$$

$$\left. - \sin m(\varphi - \varphi_0) \right] \frac{1}{2} \left[\frac{1}{m^{3/2}} - \frac{\gamma^2}{2m^{5/2}} + \right.$$

$$+ \frac{\gamma^3}{2m^3} - \frac{\gamma^4}{4m^{7/2}} +$$

$$\left. \left. + \frac{\gamma^6}{8\left(\frac{\gamma^2}{2} m^{7/2} + \gamma m^4 + m^{9/2} \right)} \right] \right\}.$$

The convergence of the remainder after separating out the singularity will be of order $O(1/m^4)$, and the constant is

$$C = \pi q_0 \varphi_0 \left\{ 2 \sum_{m=1}^{\infty} \sin^2 m\varphi_0 \times \right.$$

$$\times \left[\frac{1}{m^2} - \frac{\gamma}{2m^{5/2}} + \frac{\gamma^3}{4m^{7/2}} - \right.$$

$$\left. \left. - \frac{2\gamma^4 m^{1/2} + \gamma^5}{8m^{9/2} + 8\gamma m^4 + 4\gamma^2 m^{7/2}} \right] \right\}^{-1}.$$

In grinding practice, most often the parameter $\gamma = 0.03$ to 1.3, and the angle of contact $\varphi_0 = 0.5$ to 3°. For these ranges of variation of γ and φ_0, we can consider that

$$U(\varphi) = C \left[2\varphi_0 + \right.$$

$$+ 2\gamma \sum_{m=1}^{\infty} \sin m\varphi_0 \sin m\varphi \frac{1}{2} \left(\frac{1}{m^{3/2}} - \frac{\gamma}{m^2} + \right.$$

$$\left. + \frac{\gamma^2}{2m^{5/2}} - \frac{\gamma^4}{4m^{7/2}} \right) + \sin m\varphi_0 \cos m\varphi \frac{1}{2} \left(\frac{1}{m^{3/2}} - \right.$$

$$\left. \left. - \frac{\gamma^2}{2m^{5/2}} + \frac{\gamma^3}{2m^3} - \frac{\gamma^4}{4m^{7/2}} \right) \right],$$

and $C = q_0/2\pi\alpha$, since the discarded terms give an error of ~ 1% for $\gamma < 1$, and for $\gamma > 1$ the error is 4 to 5% in calculating T_{max}, the maximum temperature in the contact zone.

We should also take account of the fact that we assumed $\alpha_k = 0$ in condition (4); if we assume α_k, according to [1, 2], then calculations show that the difference is less than 1.5% because of the smallness of the contact angle. Therefore, if $0 \le \alpha_k \le \alpha$, we can take $\alpha_k = \alpha$. Then $C = q_0/2\pi\alpha$, also for $\alpha_k = 0$. The slowly convergent series of the form

$$\sum_{m=1}^{\infty} \frac{\cos m\varphi}{m^{\nu}} \text{ and } \sum_{m=1}^{\infty} \frac{\sin m\varphi}{m^{\nu}}$$

can be calculated using the integral representation of [5]:

$$\sum_{m=1}^{\infty} \frac{\cos m\varphi}{m^{\nu}} = \frac{1}{\Gamma(\nu)} \int_{0}^{\infty} \frac{t^{\nu-1} e^{-t} (\cos\varphi - e^{-t})}{1 - 2\cos\varphi\, e^{-t} + e^{-2t}}\, dt,$$

$$\sum_{m=1}^{\infty} \frac{\sin m\varphi}{m^{\nu}} = \frac{1}{\Gamma(\nu)} \int_{0}^{\infty} \frac{t^{\nu-1} e^{-t} \sin\varphi}{1 - 2\cos\varphi\, e^{-t} + e^{-2t}}\, dt.$$

Table 1 gives results of calculation of component surface temperatures for various values of the parameters γ, corresponding to the case of grinding of type 45 carbon steel and heat-resistant EI 437 alloy under the initial conditions of [2, 4].

A determination of the amount of heat passing into the component was made by the ETA method [3] on an EGDA 9/60 electronic integrator. The maximum contact temperature was evaluated also by means of electrical modeling. The discrepancy from the calculated data was 7 to 9%.

The discrepancy from the calculated data of [2] was 9 to 15% in calculating the maximum temperature in the contact zone, and 300 to 400% in calculating T_{min}, i.e., the temperatures remaining at the component surface at the end of the cycle (the beginning of transition to the cutting zone). In addition, the above calculation makes it possible to determine $T_{av} = q_0\varphi_0/\alpha\pi$, the average residual temperature, i.e., the steady temperature of the component.

NOTATION

T is the temperature; ω is the angular velocity; r, φ_1 are coordinate variables; a is the thermal diffusivity; λ is the thermal conductivity; R is the component radius; $2\varphi_0$ is the contact angle; α_k is the heat-transfer coefficient in the contact zone; α is the heat transfer in the out-of-contact zone; $\Gamma(\nu)$ is the gamma function.

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North-West Polytechnical Correspondence Institute, Leningrad